Multirotor Aircraft: Modelling, State Estimation and Control

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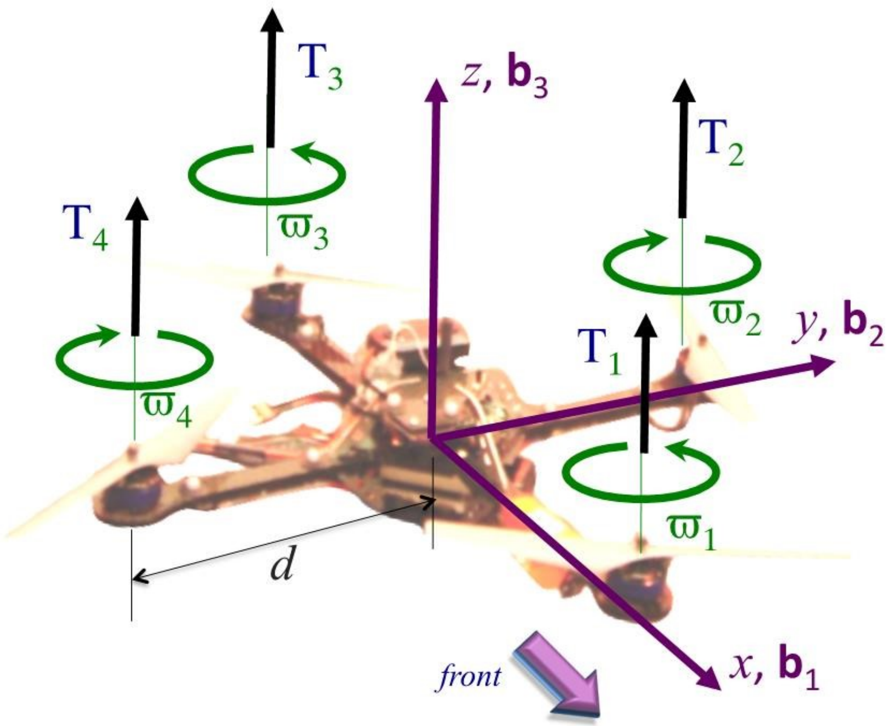
Quadrotor aerial vehicle is the most flexible and adaptable platforms for undertaking aerial research.

- Small and safe.
- Operates well in indoor laboratory environment.
- Many manufacturers but fairly uniform functionality.
- Can carry sensor and computer payload to function autonomously – at least for short time.
- Capable of hover flight or forward flight.
- Sensing and control problems are ubiquitous for aerial robotics.

Small scale research quadrotors and larger commercial vehicles use identical control code.

Part I: Modelling



A quadrotor consists of 4 individual rotors attached to a rigid cross airframe.

Quadrotor control is achieved by differential control of the thrust generated by each rotor.

- **Heave** (total thrust) is the sum of thrust generated by each rotor.
- **Pitch** and **roll** are obtained by differential thrust along the NS axis or EW axis.
- **Yaw** control is obtained by differential control of the NS rotors compared to the EW rotors such that the total thrust is constant.

A quadrotor is under-actuated and the remaining degrees of freedom, translational velocity in the horizontal plane, must be controlled through the system dynamics.

N - rotor vehicles

The conceptual idea of a quadrotor is straightforward to extend to multirotor vehicles.

The advantage of these configurations is safety and and redundancy rather than aerodynamic efficiency.

Rigid body dynamics of quadrotor

velocity

gravity

position

thrust

drag

$$\dot{\xi} = v,$$

$$m\dot{v} = mge_3 - TRe_3 - H,$$

rotation matrix

$$\dot{R} = R\Omega_\times,$$

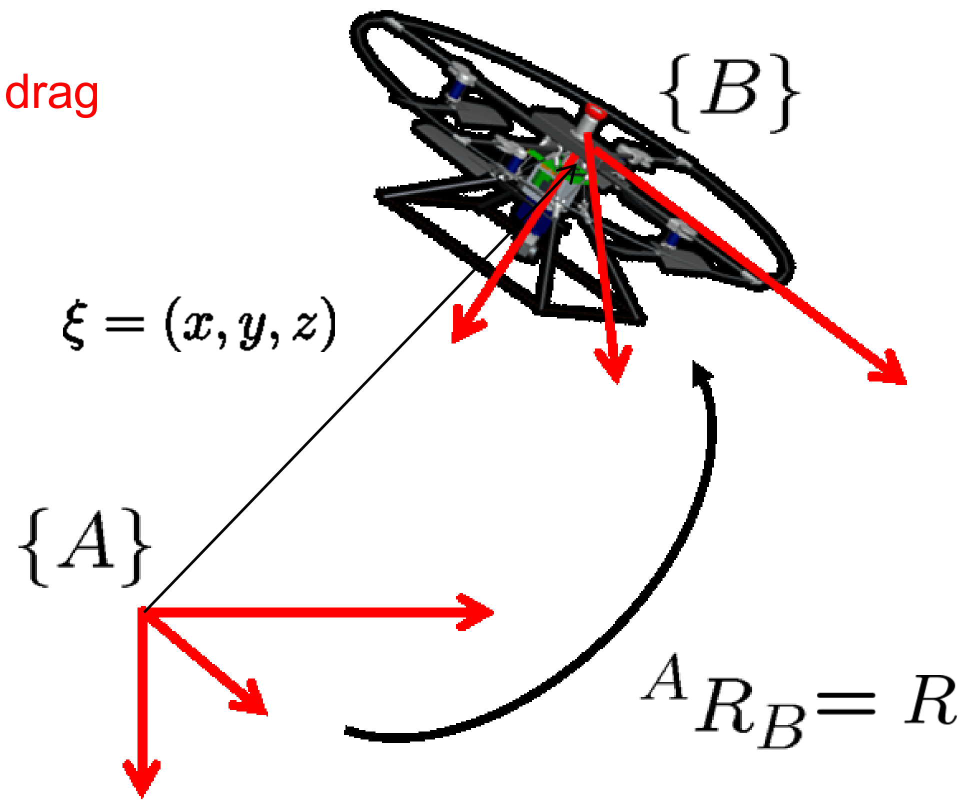
$$I\dot{\Omega} = -\Omega \times I\Omega + \tau,$$

inertia tensor

torque

angular velocity

$$\boxed{\Omega_\times = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix}}$$



Orientation matrix $^A R_B \in SO(3)$.

$$\dot{\xi} = v$$
$$m\dot{v} = mge_3 + TRe_3$$
$$\dot{R} = R\Omega_\times$$
$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \tau.$$

Rigid-body dynamics in homogeneous coordinates

$R^T R = I$

$$\dot{\xi}_1 = v_1,$$
$$\dot{\xi}_2 = v_2,$$
$$\dot{\xi}_3 = v_3,$$
$$m\dot{v}_1 = T(c_\psi s_\theta c_\phi + s_\psi s_\phi)$$
$$m\dot{v}_2 = T(c_\psi s_\theta s_\phi - s_\psi c_\phi)$$
$$m\dot{v}_3 = mg + T(c_\psi c_\theta)$$

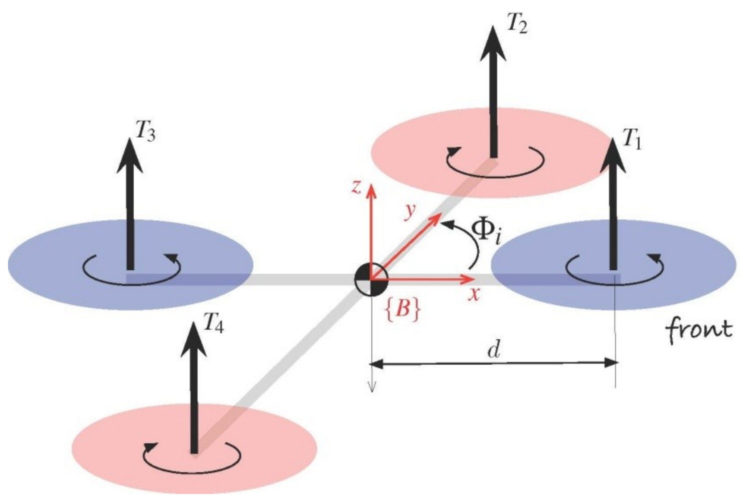
$$\dot{\phi} = \frac{s_\psi}{c_\theta}\Omega_2 + \frac{c_\psi}{c_\theta}\Omega_3$$
$$\dot{\theta} = c_\psi \Omega_2 - s_\psi \Omega_3$$
$$\dot{\psi} = \Omega_1 + t_\theta s_\psi \Omega_2 + t_\theta c_\psi \Omega_3$$
$$\mathbf{I}_1\dot{\Omega}_1 = (\mathbf{I}_2 - \mathbf{I}_3)\Omega_2\Omega_3 + \tau_1$$
$$\mathbf{I}_2\dot{\Omega}_2 = (\mathbf{I}_3 - \mathbf{I}_1)\Omega_1\Omega_3 + \tau_2$$
$$\mathbf{I}_3\dot{\Omega}_3 = (\mathbf{I}_1 - \mathbf{I}_2)\Omega_1\Omega_2 + \tau_3$$

Rigid-body dynamics in local generalised coordinates.

$$\dot{x} = f(x, u)$$

The aerodynamics of rotors was extensively studied during the mid 1900's with the development of manned helicopters and detailed models of rotor aerodynamics are available in the literature.

Much of the detail in these aerodynamic models is useful for the design of rotor systems where the whole range of parameters, rotor geometry, profile, hinge mechanism, and much more, are fundamental to the design problem.

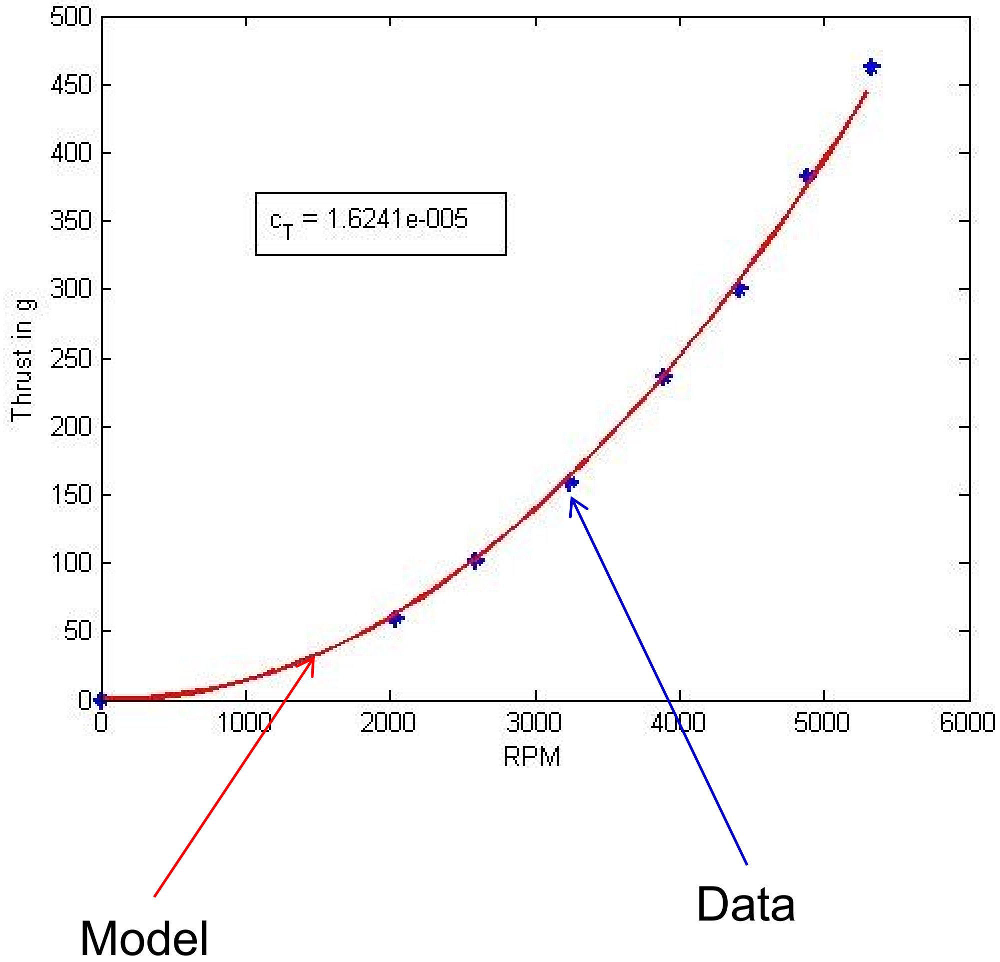


For a typical robotic quadrotor vehicle the rotor design is a question of choosing one of among five or six available rotors and motors from the hobby shop. Most of the complexity of aerodynamic modeling is best ignored.

The thrust c_T coefficient for a given rotor motor system is best identified using static thrust tests.

Lumped parameter model

$$T = c_T \varpi^2$$



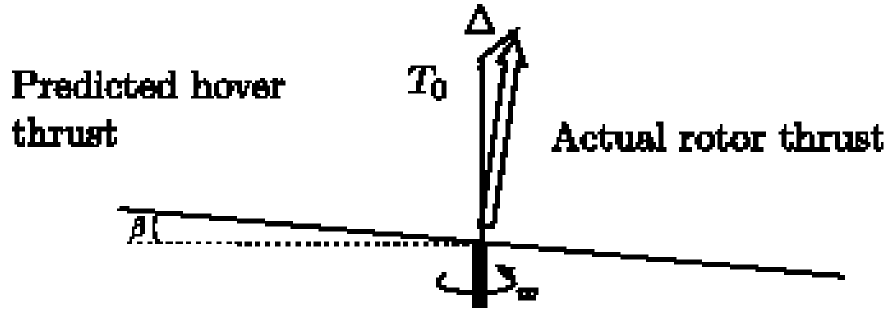
The total thrust at hover

$$T_0 = \sum_{i=1}^{4} |T_i| = c_T \left(\sum_{i=1}^{4} \varpi_i^2 \right)$$

The hover thrust is the the primary component of the exogenous force

$$F = -T_0 e_3 + \Delta$$

where Δ comprises secondary aerodynamic forces that are induced when the assumption that the rotor is in hover is violated.

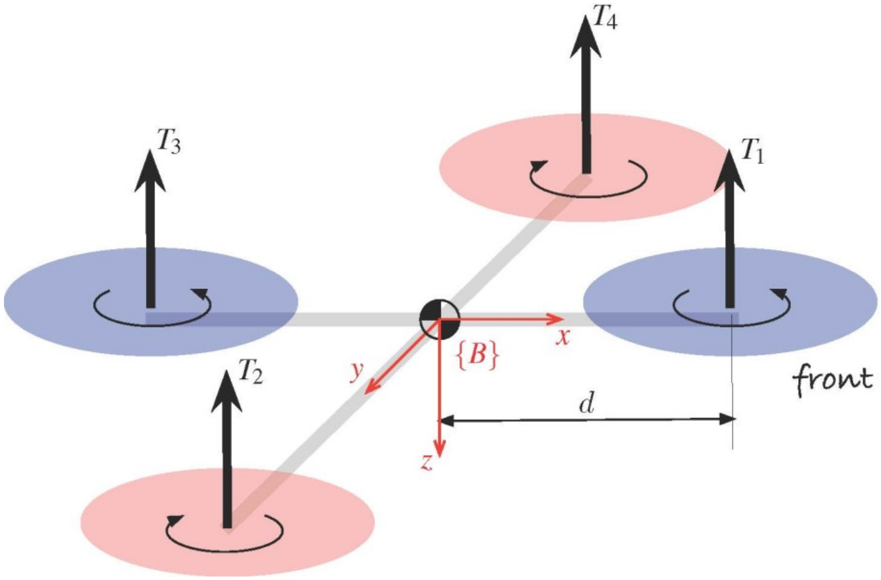


We will look at one or two of the secondary aerodynamic effects since they effect secondary stability.

But we will base the control design on the primary model.

The net moment arising from the aerodynamics are

$$\tau_x = -c_T d_i(\varpi_4^2 - \varpi_2^2)$$
$$\tau_y = c_T d_i(\varpi_1^2 - \varpi_3^2)$$
$$\tau_z = c_Q(\varpi_1^2 - \varpi_2^2 + \varpi_3^2 - \varpi_4^2)$$



We can write this in matrix form

$$\begin{pmatrix} T_0 \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \underbrace{\begin{pmatrix} c_T & c_T & c_T & c_T \\ 0 & -dc_T & 0 & dc_T \\ dc_T & 0 & -dc_T & 0 \\ c_Q & -c_Q & c_Q & -c_Q \end{pmatrix}}_{\Gamma} \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix}$$

In the control design, then the desired thrust and moments we can be solved for the required rotor speeds using the inverse of the constant matrix Γ.

In order for the vehicle to hover, one must choose suitable ϖ_i by inverting Γ such that $\tau = 0$ and $T_0 = mg$.

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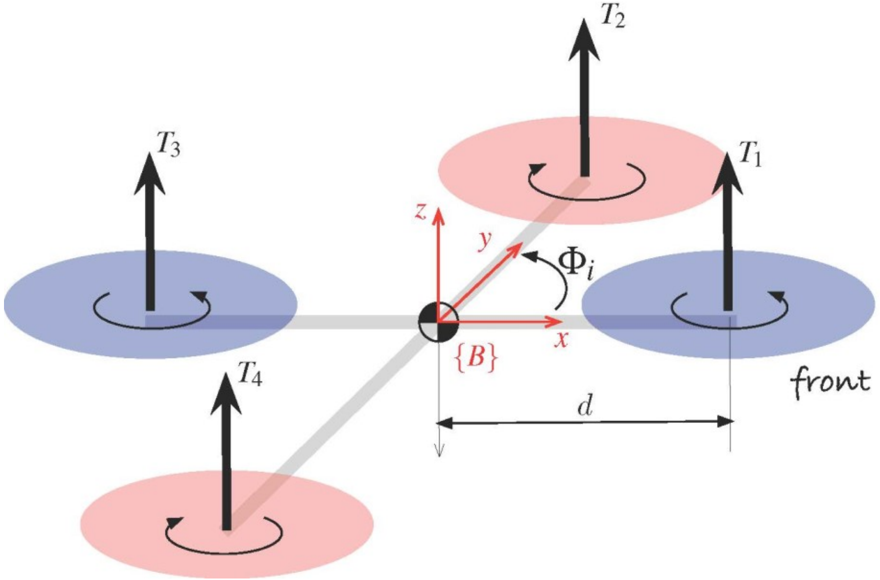
$$T_0 = \sum_{i=1}^{N} |T_i| = c_T \left(\sum_{i=1}^{N} \varpi_i^2 \right)$$

$$\tau_x = -c_T \sum_{i=1}^{N} d_i \sin(\Phi_i) \varpi_i^2$$

$$\tau_y = c_T \sum_{i=1}^{N} d_i \cos(\Phi_i) \varpi_i^2$$

$$\tau_z = c_Q \sum_{i=1}^{N} \sigma_i \varpi_i^2$$



There are a number of secondary aerodynamic effects associated with quadrotors.

Horizontal forces

- Blade flapping
- Parasitic drag
- Aerodynamic drag

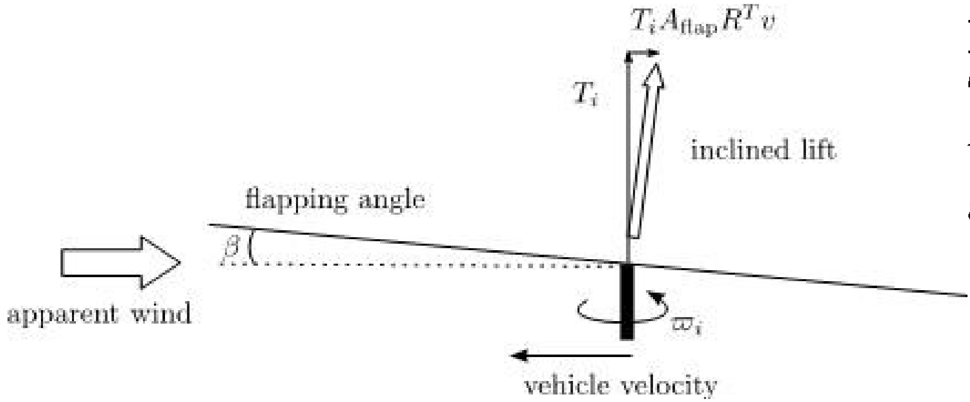
Vertical Forces

- Ground effect
- Inflow damping
- Translational lift

Other aerodynamic effects

- Vortex states
- Flow variation due to local environment

Horizontal aerodynamic forces

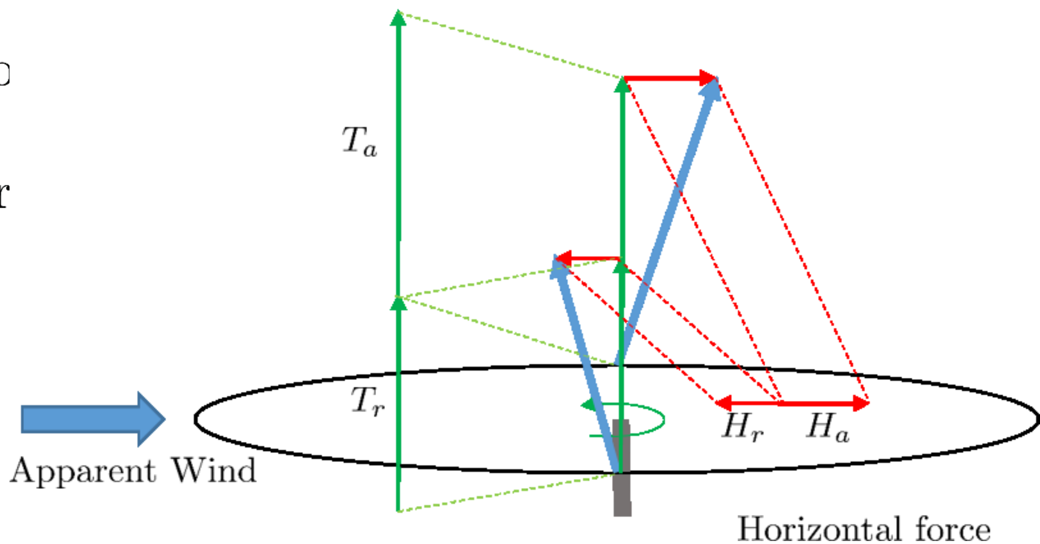


Blade flapping:
The flexibility of the blades allows the tip-path-plane to tilt back to balance aerodynamic forces.
The rotor thrust is tilted back and generates a Horizontal force

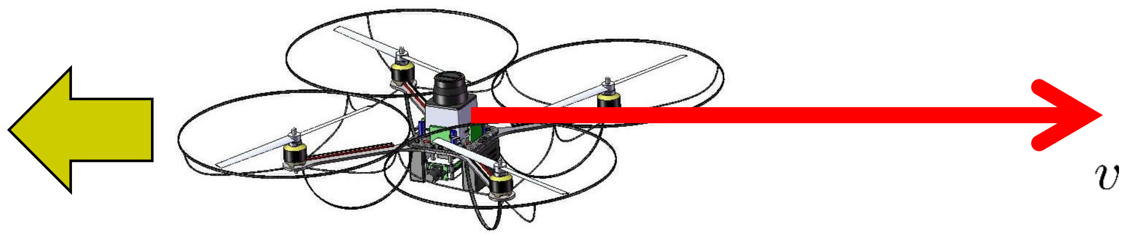
$$H \propto -v_{\mathrm{h}} \frac{T}{\varpi r}$$

Parasitic drag:
The advancing rigid blade generates mo zontal force $\propto -(\varpi R + v_{\mathrm{h}})^2$.
The retreating blade generates less hor force $\propto +(\varpi R - v_{\mathrm{h}})^2$.
The net horizontal force is

$$H \propto -v_{\mathrm{h}} \frac{T}{\varpi r}$$



Aerodynamic drag



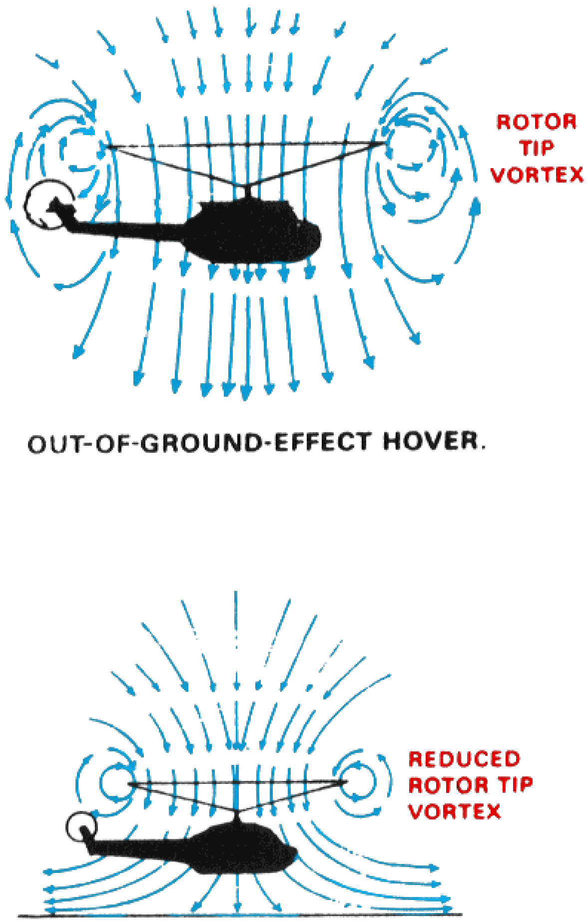
Drag force

$$\boxed{F_D \propto v_{\mathrm{h}}^2}$$

Aerodynamic drag has similar characteristics to blade flapping effects.

- Aerodynamic drag is quadratic in velocity: It is negligible at low speeds and dominates at high speed.
- Blade flapping is linear with velocity: It is dominant at low speeds and negligible at high speed.

Ground effect

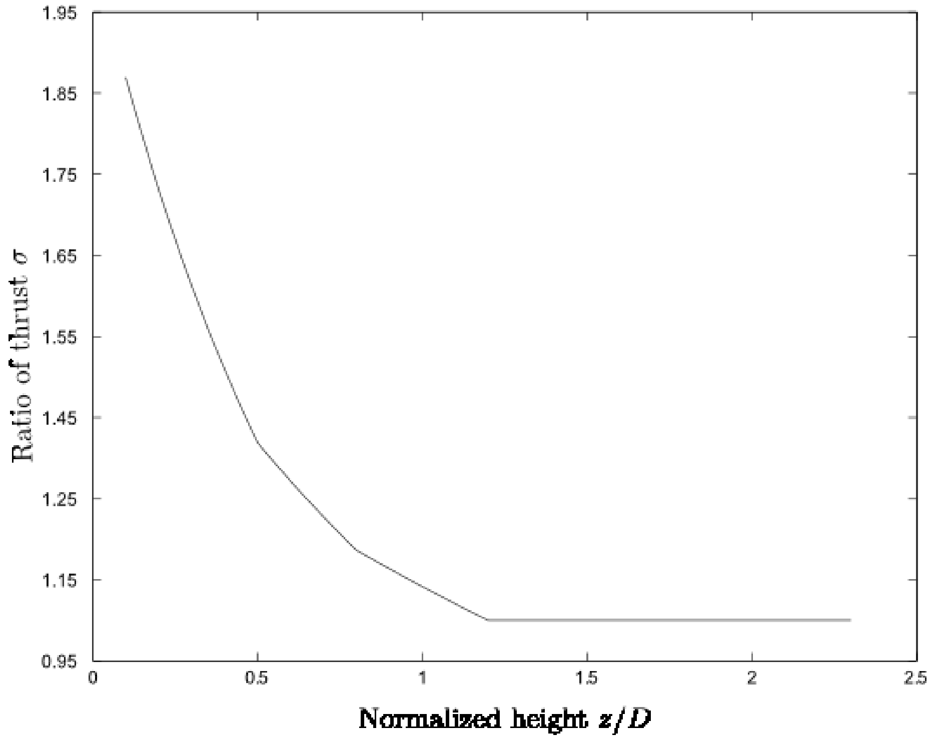


OUT-OF-GROUND-EFFECT HOVER.

FIGURE 2-38. IN-GROUND-EFFECT HOVER.

With ground effect the actual thrust generated is higher than the free air thrust

$$T_{\text{actual}} = \sigma T_{\text{free}}$$



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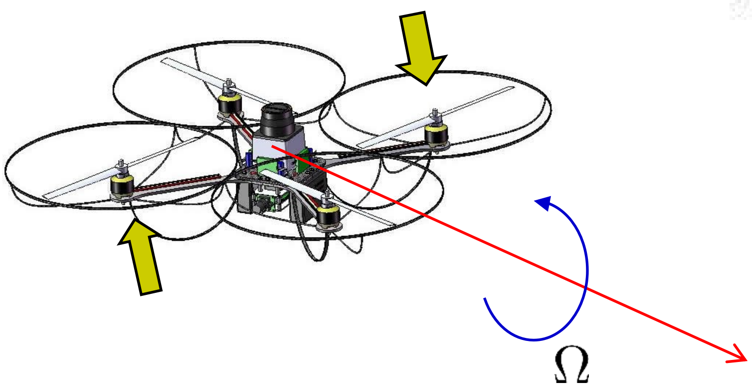
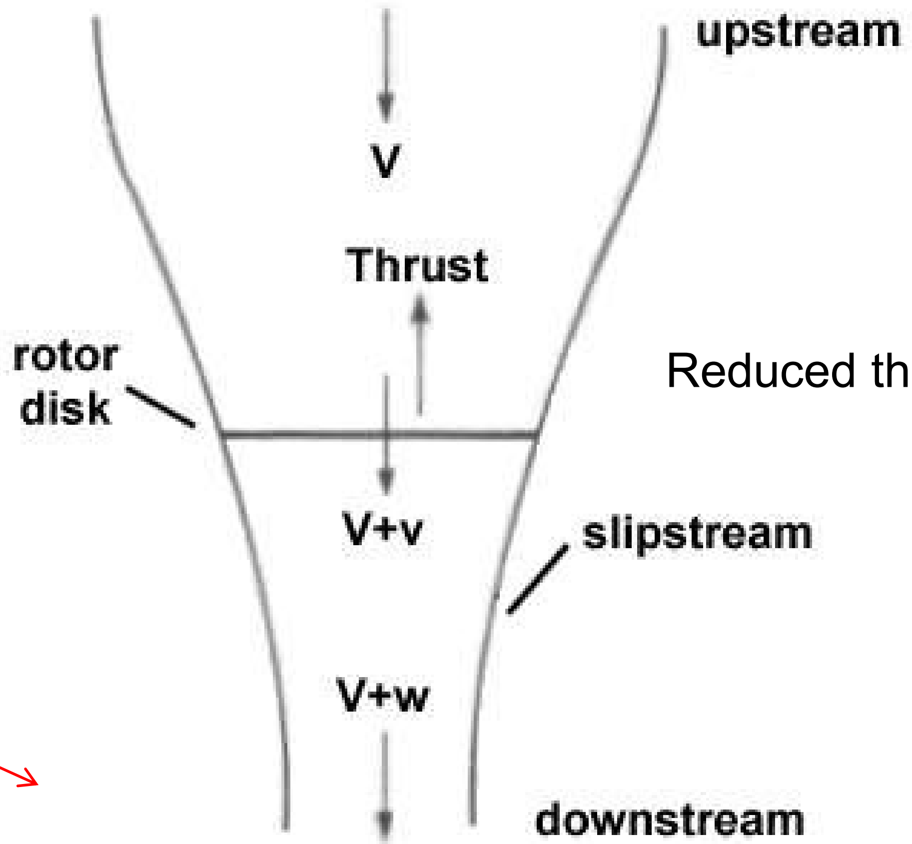
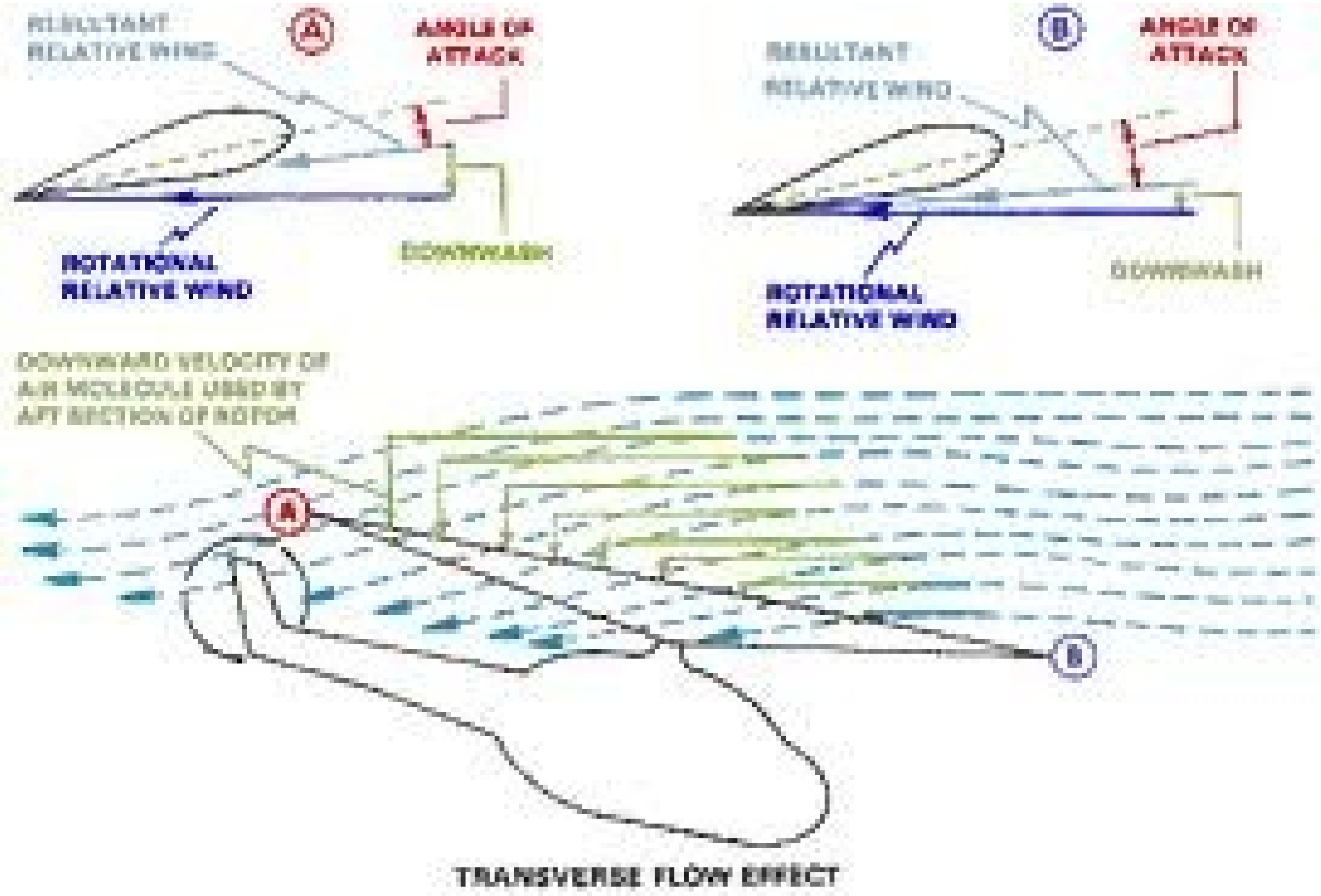
Inflow damping





Reduced thrust

Translational lift



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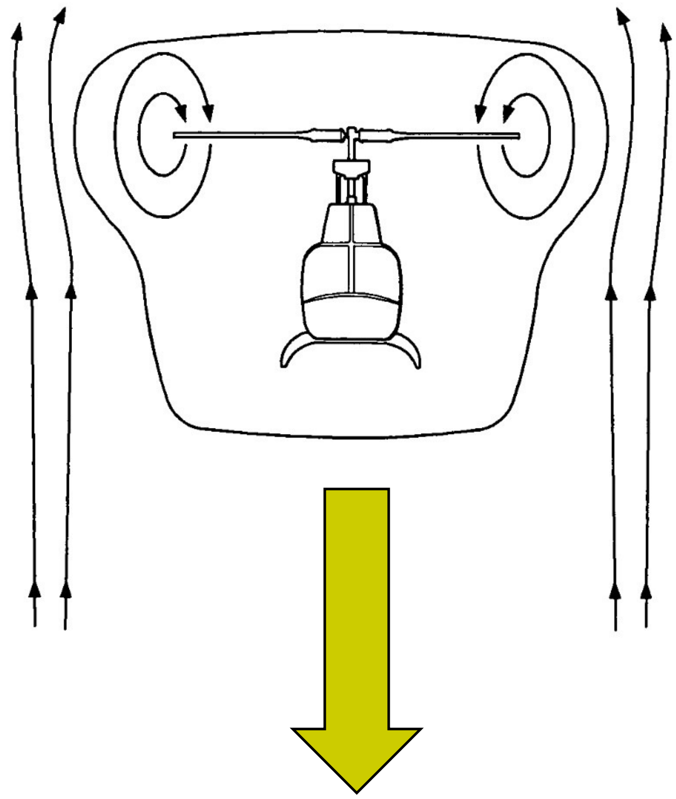
A vortex state in a full sized helicopter occurs when the tip vortex grows to create a closed air cell in which the helicopter is trapped.   The helicopter will fall to the ground unless the pilot takes action.

Vortex state is generally caused in vehicles carrying heavy payloads and descending slowly. For this reasons, most helicopters do not directly descend in hover, they will always fly in at an angle.



Part II: State Estimation

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⋆ The state estimation problem for small quadrotor vehicles is a fundamental component of the avionics.

⋆ Effective algorithms are simple, robust, low computational-complexity.

⋆ Complementary oberservers were an enabling technology from 2005-2010 and are still an observer of choice for many groups, although sensor quality has now improved to the point where EKF and other more sophisticated algorithms will work.

Measurements in a typical Avionics Suite

Gyroscope:	$\Omega_{\mathrm{IMU}} = \Omega + b_\Omega + \eta$
Accelerometer:	$a_{\mathrm{IMU}} = R^\top(\dot{v} - g\mathbf{e}_3) + b_a + \eta_a$
Magnetometer:	$m_{\mathrm{IMU}} = R^\top m + B_m(i_1, \ldots, i_4) + \eta_b$
Barometer:	$z_{\mathrm{IMU}} = z - z_0 + b_z + \eta_z$
GPS:	$\xi_{\mathrm{GPS}} = \xi + \eta_\xi$

⋮

Vision

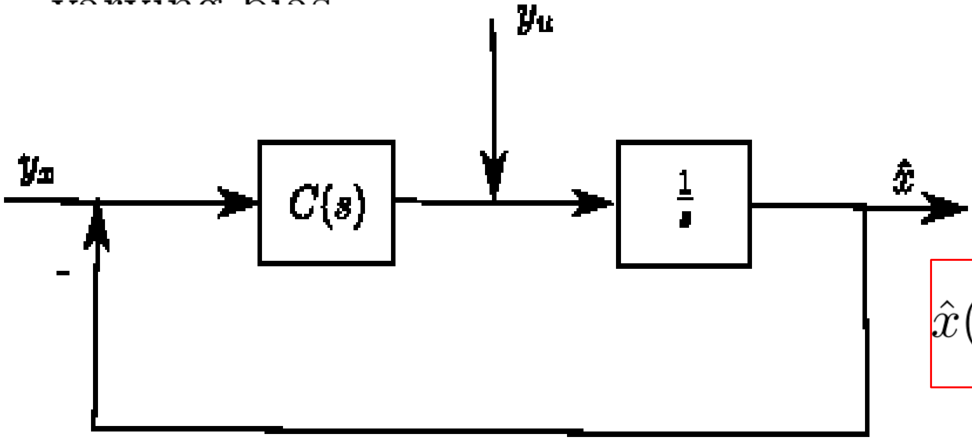
Complementary filters are particularly well suited to fusing low bandwidth position measurements with high band width rate measurements for first order kinematic systems.

$$\dot{x} = u.$$

with typical measurement characteristics

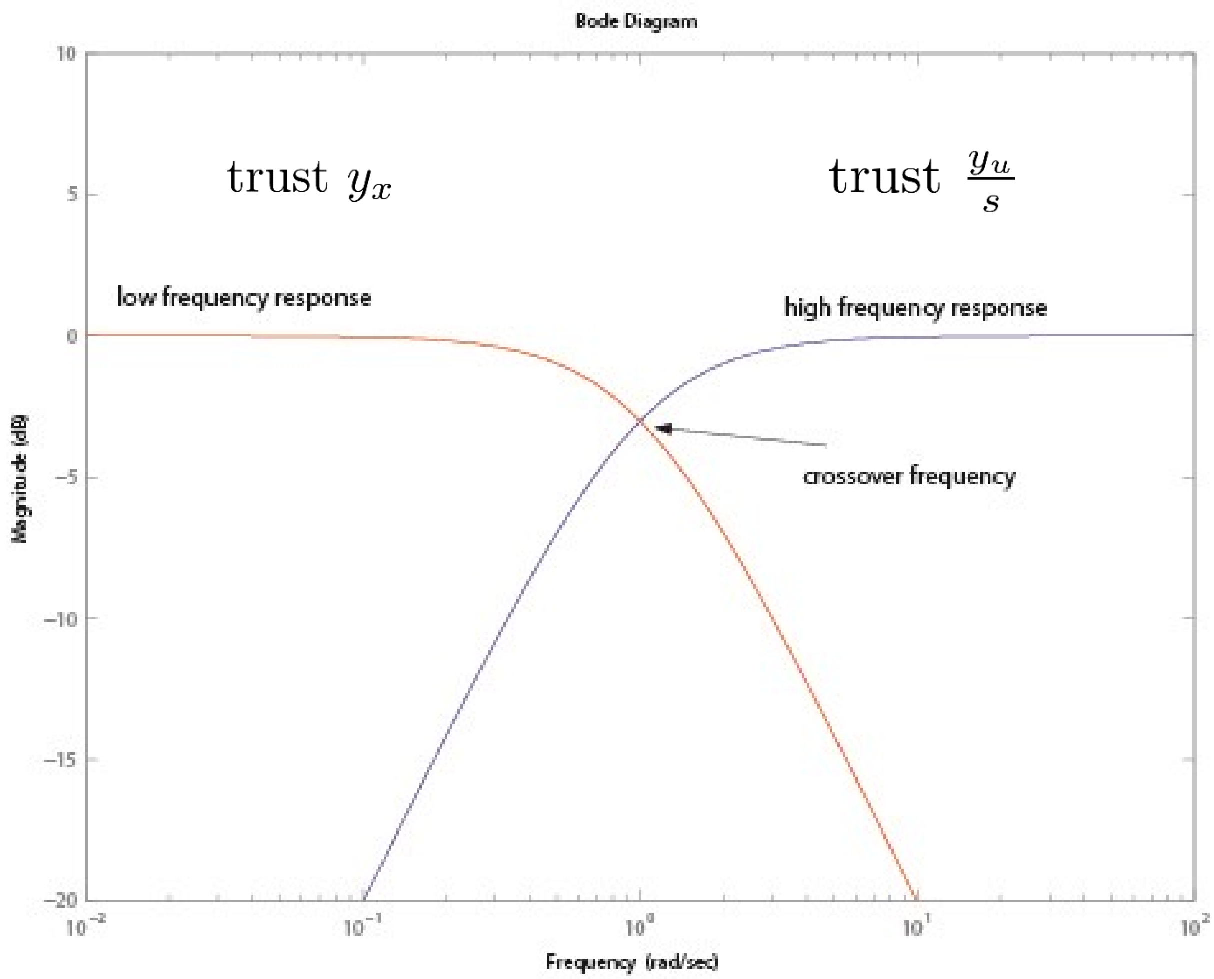
$$y_x = L(s)x + \mu_x, \quad y_u = u + \mu_u + b(t)$$

$L(s)$ is low pass sensor characteristic, μ is noise and $b(t)$ a slowly time-varying bias



$$\boxed{\hat{x}(s) = \frac{C(s)}{s + C(s)} y_x(s) + \frac{s}{C(s) + s} \frac{y_u(s)}{s}}$$

Frequency response of complementary filter



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Use the magnetometer and accelerometer to provide rough estimates of attitude. Use the Gyrometer for the angular velocity

$$\boxed{m_{\mathrm{IMU}} = R^{\top} \mathring{m}}$$

Consider only the low-frequency part of the signal. In particular, if $\dot{v} \approx 0$

$$\boxed{\frac{H}{m} = g\mathbf{e}_3 - \frac{T}{m} R\mathbf{e}_3}$$

$$\boxed{\dot{v} = g e_3 - \frac{T}{m} R e_3 - \frac{H}{m},}$$

Substitute dynamics into accelerometer measurements

$$a_{\mathrm{IMU}} = R^{\top}\left(-\frac{T}{m} R\mathbf{e}_3 - \frac{H}{m}\right)$$

$$\boxed{a_{\mathrm{IMU}} = R^{\top}(\dot{v} - g\mathbf{e}_3)}$$

Substituting yields

$$\boxed{\bar{a}_{\mathrm{IMU}} \approx -g R^{\top} \mathbf{e}_3}$$

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The proposed observer on SO(3) is

$$\boxed{\dot{\hat{R}} = \hat{R}\left(\Omega - k(\hat{y} \times y)\right)_\times, \qquad \hat{R}(0) = I_3}$$

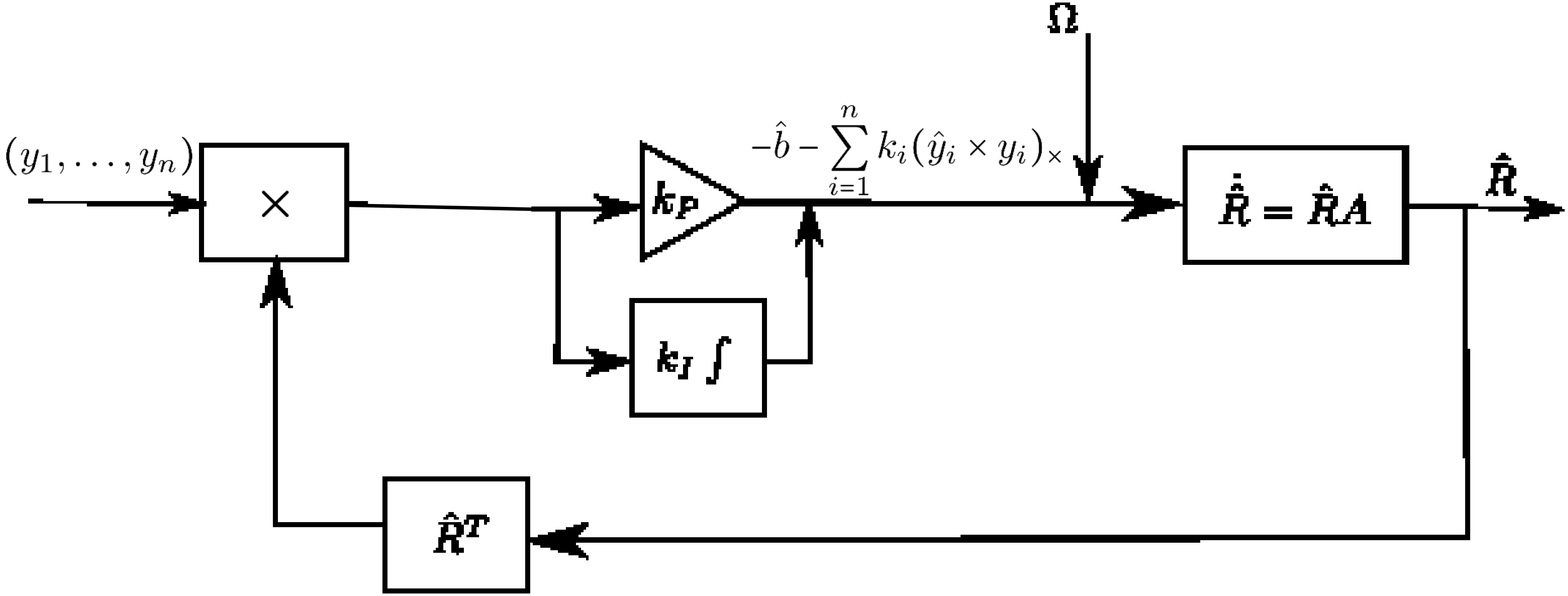
In quaternion representation one has

$$\dot{\hat{q}} = \frac{1}{2}\hat{q} \otimes \mathbf{p}(\Omega_y - k(\hat{y} \times y))$$
$$\hat{y} = \mathbf{P}^\dagger\left(\hat{q} \otimes \mathbf{P}(y_0) \otimes \hat{q}^{-1}\right)$$

For multiple measurement one has

$$\dot{\hat{R}} = \hat{R}\left(\Omega_\times - \sum_{i=1}^{n} k_i(\hat{y}_i \times y_i)_\times\right), \qquad \hat{R}(0).$$

$$y_1 = \frac{a_{\mathrm{IMU}}}{|a_{\mathrm{IMU}}|}$$
$$\hat{y}_1 = \hat{R}^\top \mathbf{e}_3$$
$$\vdots$$
$$y_2 = m_{\mathrm{IMU}}$$
$$\hat{y}_2 = \hat{R}^\top \mathring{m}$$



Ω
$(y_1, \ldots, y_n)$
$\times$
k_P
$-\hat{b} - \sum_{i=1}^{n} k_i (\hat{y}_i \times y_i)_\times$
$k_I \int$
$\dot{\hat{R}} = \hat{R}A$
$\hat{R}$
$\hat{R}^T$

Velocity aided attitude

It is also possible to develop a complementary observer that does a good job of estimating linear velocity and attitude at the same time.

$$\dot{\hat{v}} = ge_3 + \hat{R}a - k_v(\hat{v} - v)$$

$$\dot{\hat{R}} = \hat{R}(\Omega - k_m m \times \hat{m} - \underbrace{k_a a \times \hat{R}^\top(\hat{v} - v)}_{\boxed{k_a(a \times \hat{R}^\top \tilde{v})_\times}})_\times$$

Either v is measured directly by doppler GPS or one uses a coupled filter

$$\dot{\hat{\xi}} = \hat{v} - k_1(\hat{\xi} - \xi_{\text{GPS}})$$

$$\dot{\hat{v}} = \hat{R}a - g\mathbf{e}_3 - k_2(\hat{\xi} - \xi_{\text{GPS}})$$

The complicated term in this observer design is the acceleration innovation for attitude

$$\Delta = k_a(a \times \hat{R}^\top(\hat{v} - v))_\times = k_a(a \times \hat{R}^\top \tilde{v})_\times$$

Consider the $\tilde{v}$ error kinematics

$$\boxed{\dot{\tilde{v}} = \hat{R}a - k_v\tilde{v} - Ra = (I - R\hat{R}^\top)\hat{R}a - k_v\tilde{v},}$$

Think of the $\tilde{v}$ as the output of a linear system and take just the low-frequency signal

$$\tilde{v} = \frac{1}{s + k_v}(I - R\hat{R}^\top)\hat{R}a \approx \frac{1}{k_v}(I - R\hat{R}^\top)\hat{R}a$$

Thus,

$$\begin{aligned}\Delta &= k_a a \times \hat{R}^\top \tilde{v} \approx \frac{k_a}{k_v} a \times \hat{R}^\top (I - R\hat{R}^\top)\hat{R}a \\ &= -\frac{k_a}{k_v} a \times \hat{R}^\top Ra = -\frac{k_a}{k_v} a \times \hat{R}^\top Ra = k_a a \times \hat{R}^\top \mathring{a} = \boxed{k_a a \times \hat{a}}\end{aligned}$$

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It is also possible to use the velocity estimate derived from the accelerometer in the body-fixed frame

$$
\begin{aligned}
a_{\mathrm{IMU}} &= -\frac{T_\Sigma}{m}\mathbf{e}_3 - \frac{T_\Sigma}{m} v_{\mathrm{h}} \\
&= -g\mathbf{e}_3 - \mu v_h
\end{aligned}
$$

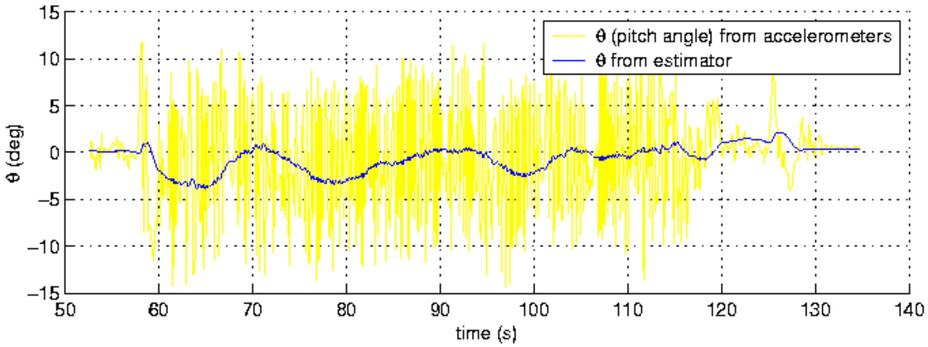
where $k > 0$ is a drag constant that needs to be identified ($k \approx 0.3$).

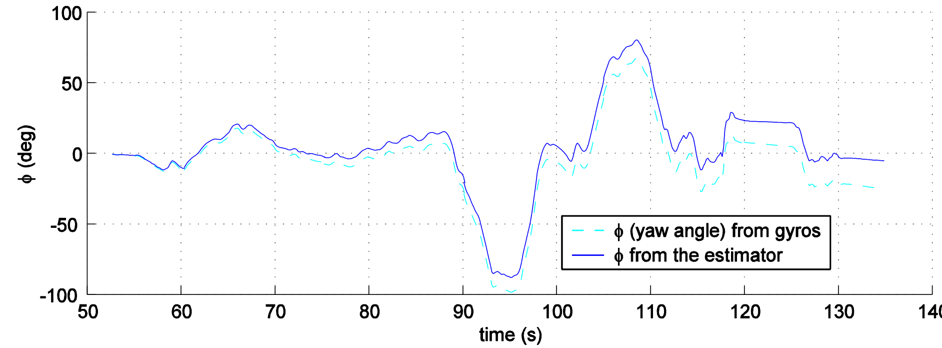
$$
v_h \approx \frac{1}{\mu}(\mathbf{e}_1^\top a_{\mathrm{IMU}}, \mathbf{e}_2^\top a_{\mathrm{IMU}})
$$

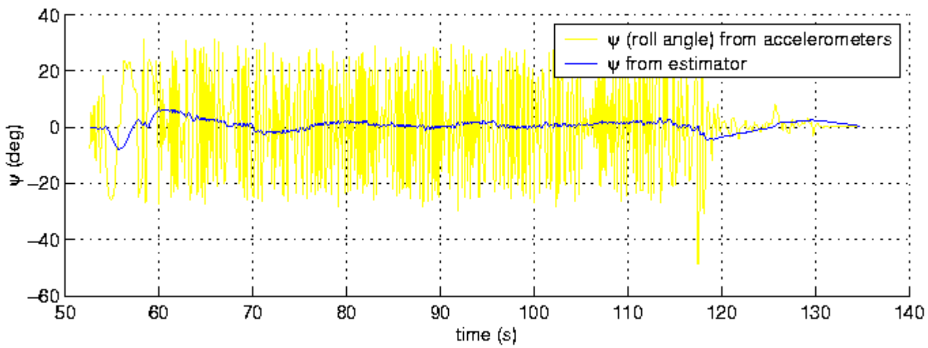
The v_z component can be estimated using the barometer in a coupled filter with the attitude.

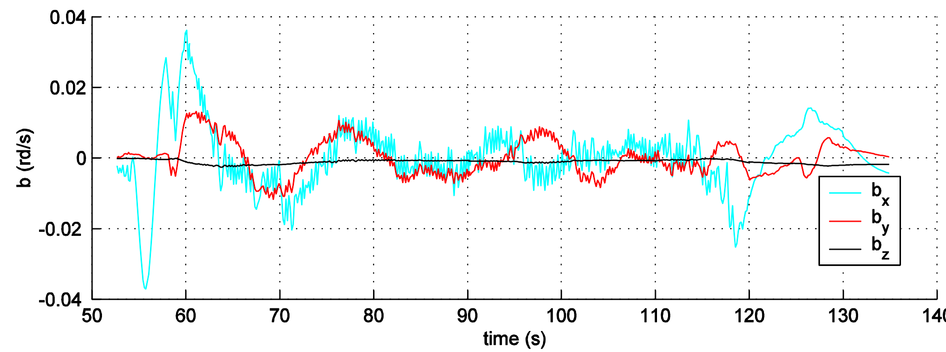
*Allibert, Mahony, Bangura, "Velocity Aided Attitude Estimation for Aerial Robotic Vehicles Using Latent Rotation Scaling", ICRA 2016 (*TuCbT3.12)

Results





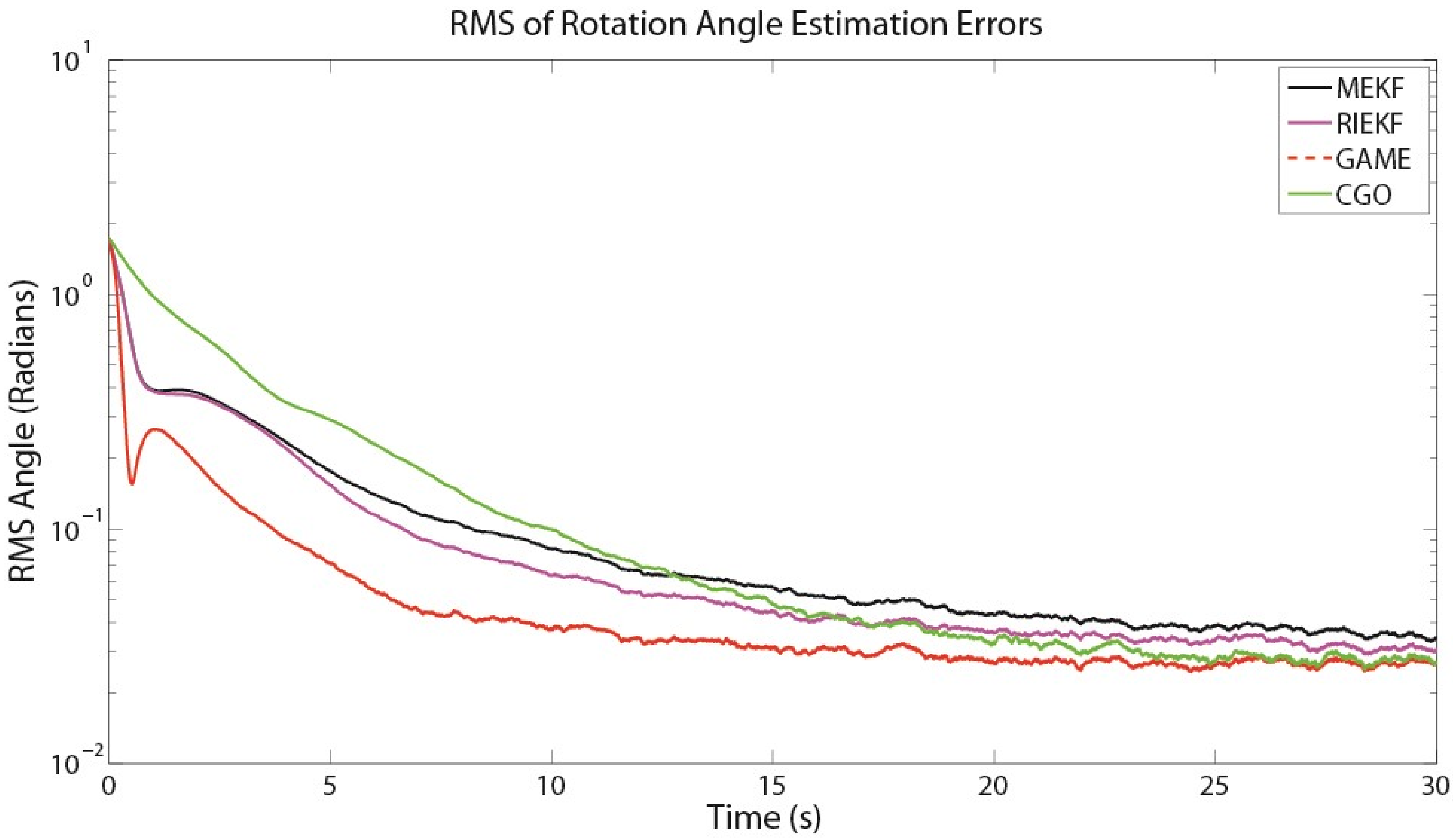




- Implementation on several UAV systems are all highly successful.

- Easily add estimation of gyro bias.
- When accelerometers are used for attitude estimate this minimises drift in yaw estimate.

Monte-Carlo comparison of performance



Monte-Carlo simulation averaged over 100 repeats of initial conditions drawn from an initial distribution. Noise characteristics typical of UAV platform. (Zamani 2012)

Part III: Control

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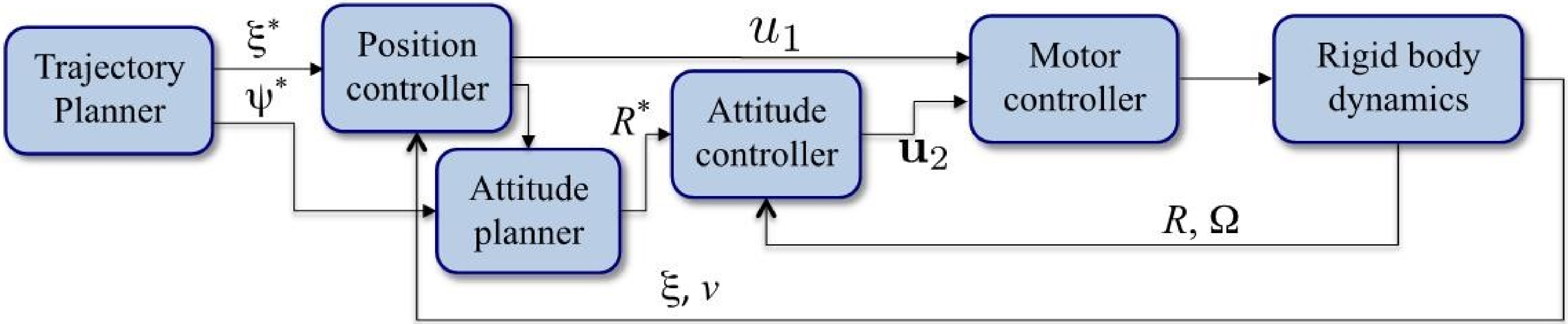
Control architecture

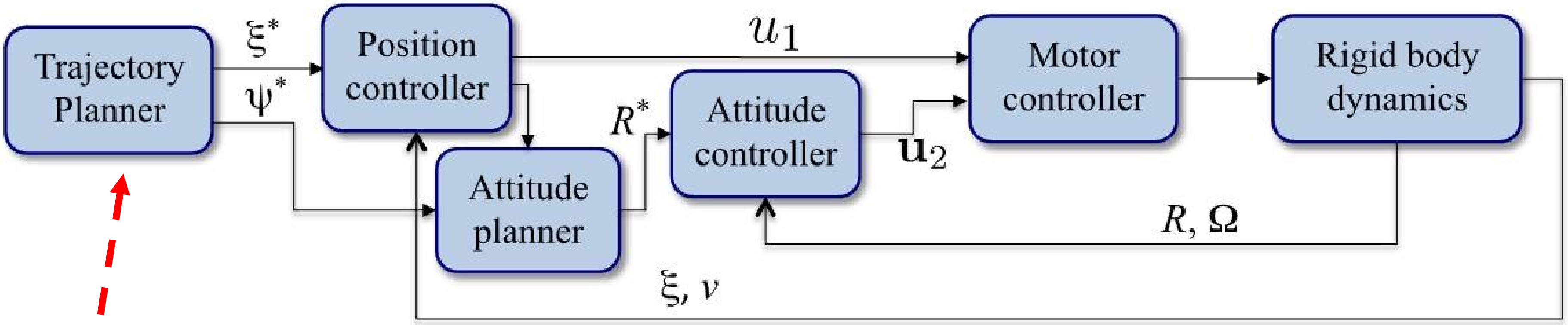


Almost all control systems for small scale aerial robotics use a hierarchical control architecture as shown above.

The key to high performance control is

- A trajectory planner that minimizes snap actuation requirements on the motors.
- Integrated attitude trajectory planning with attitude control.
- High gain robust motor control

The key constraint in the control is the motor control response, including steady state error, disturbance rejection, and limited bandwidth..



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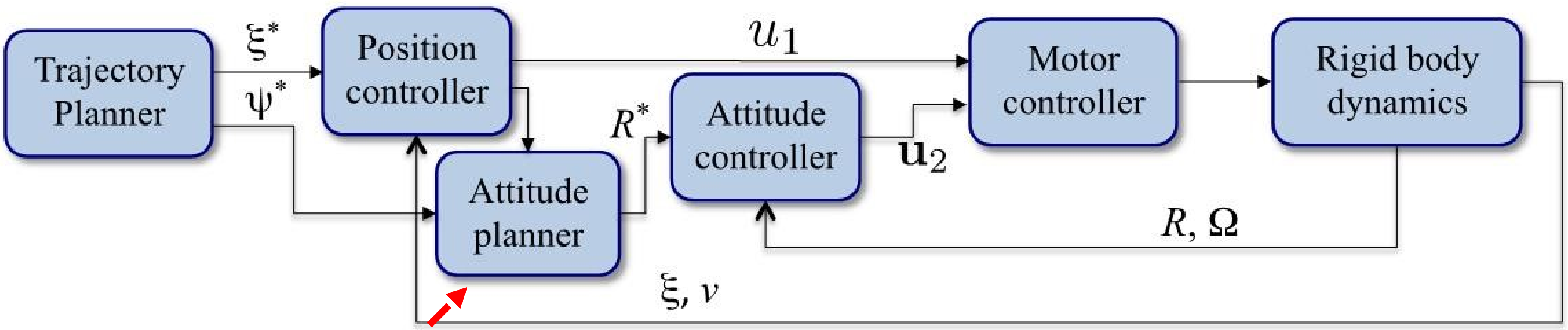

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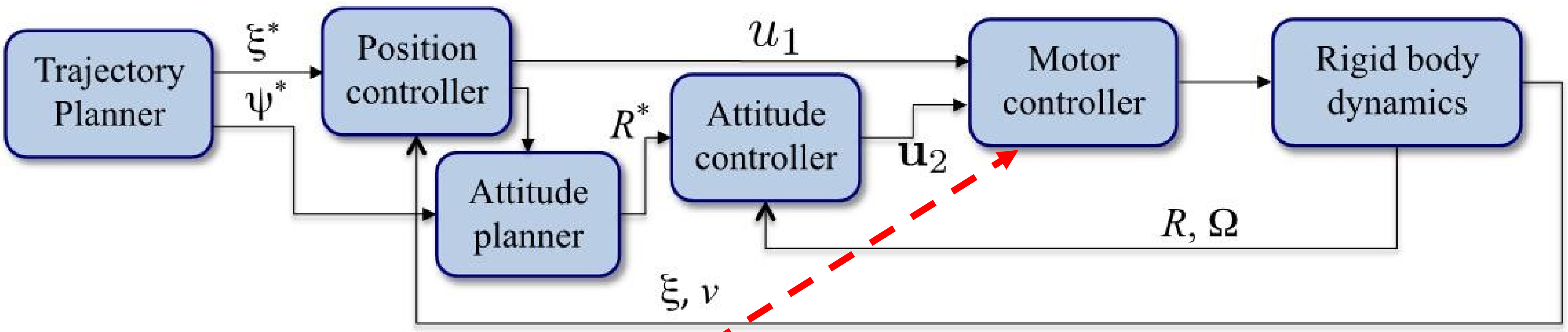

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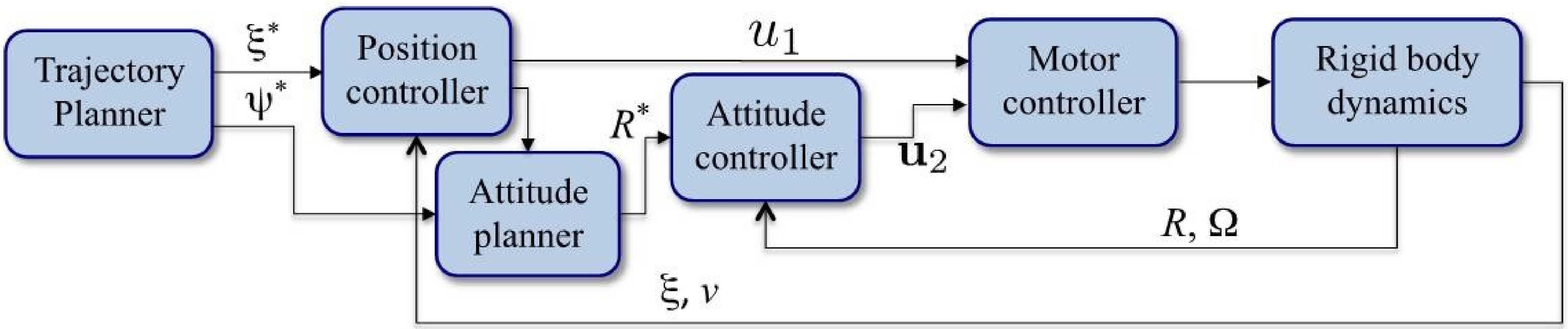

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Block diagram of system architecture

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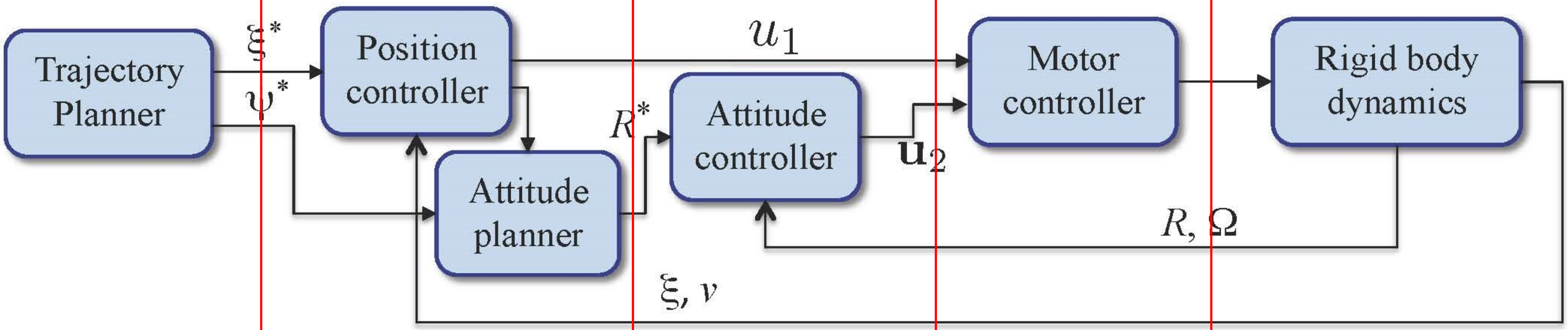
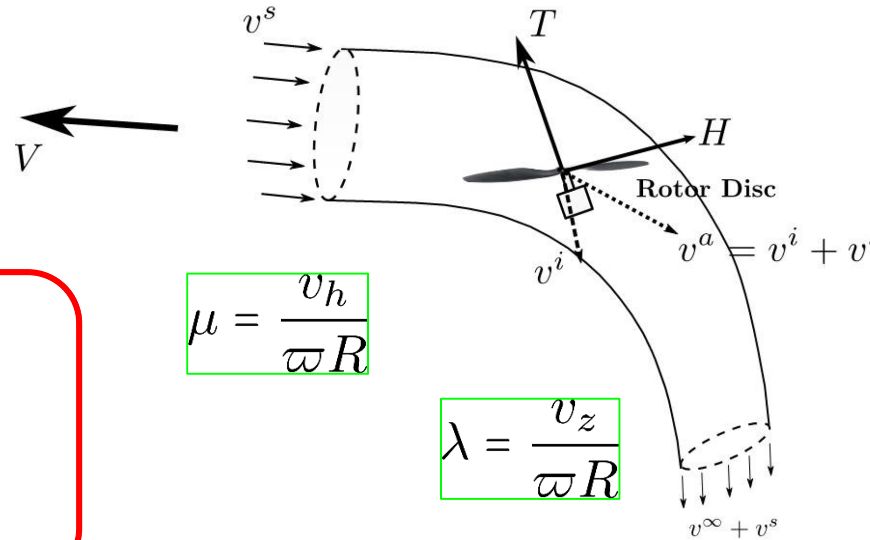


Slow	Normal	Fast	Fastest	
Inertial Navigation System		Avionics	ESC	Electronic speed controllers
Typical value 1-5Hz	Typical value 10-20Hz	Typical value 50-200Hz	Typical value 500Hz-2kHz	

Limitation is typically the motor control bus rate

Thrust depends on the local aerodynamics as well as the rotor angular velocity.



$$\boxed{\mu = \frac{v_h}{\varpi R}}$$

$$\boxed{\lambda = \frac{v_z}{\varpi R}}$$

Momentum theory

$$T = 2\rho A R^2 \varpi^2 \lambda^i \sqrt{\mu^2 + \lambda^2},$$
$$H = 2\rho A R^2 \varpi^2 \mu^i \sqrt{\mu^2 + \lambda^2},$$
$$P_a = \left(\kappa T \lambda^i + T\lambda^s + H(\kappa \mu^i + \mu^s)\right)\varpi R$$

Blade element momentum theory BEMT

$$T = \frac{1}{4} N_b \rho c_{\mathrm{tip}} R^3 \varpi^2 C_{l\alpha}\left(\theta_{\mathrm{tip}}(2 + \mu^2) - 2\lambda\right),$$
$$H = \frac{1}{2} N_b \rho c_{\mathrm{tip}} \varpi^2 R^3 \mu \left[C_{d0} + \frac{1}{2} X\right],$$
$$P_{am} = \frac{1}{4}\rho N_b c_{\mathrm{tip}} \varpi^3 R^4 C_{d0}\left(2 + 5\mu^2\right) + \left(\kappa T\lambda^i + T\lambda^s + H(\kappa\mu^i + \mu^s)\right)\varpi R,$$

Assume that $\mu^2 \approx 0$ is small with respect to λ and ignore these terms.

Define the normalised thrust and power coefficients

$$C_T = \frac{T}{\varpi^2}, \quad C_P = \frac{P}{\varpi^3},$$

The thrust equation from momentum theory yields (for constant c_4)

$$C_T = c_4 \lambda^i \lambda$$

Identify the constants in offline experiments.

Blade element momentum theory (BEMT) yields (for constants c_0, c_1, c_2, c_3)

$$C_T = c_1 \left[c_2 - \lambda \right],$$

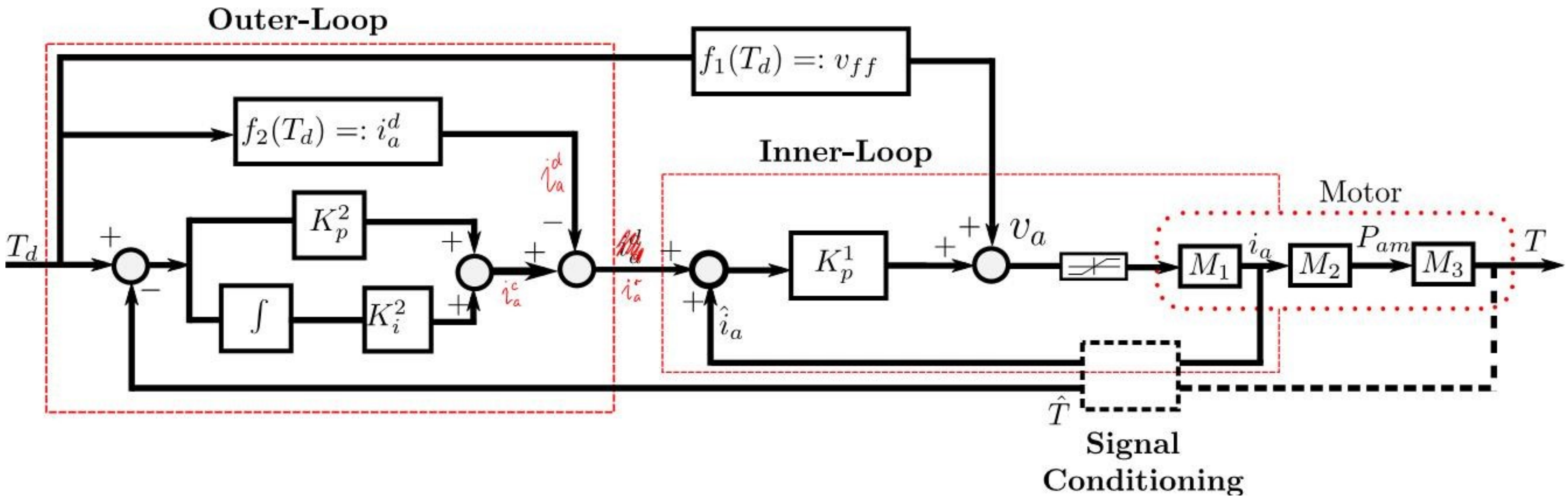
$$C_P = c_3 + C_T \left(\kappa \lambda^i + \lambda^s \right) c_0.$$

3 equations in 3 unknowns C_T, λ^i and λ^s.

C_P is measured electrical power

Computes C_T in real time on the electronic speed controller

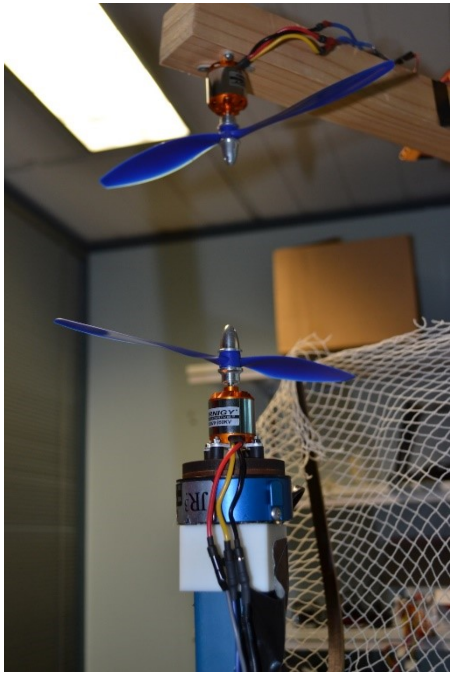
Motor-Rotor control loop design on ESC



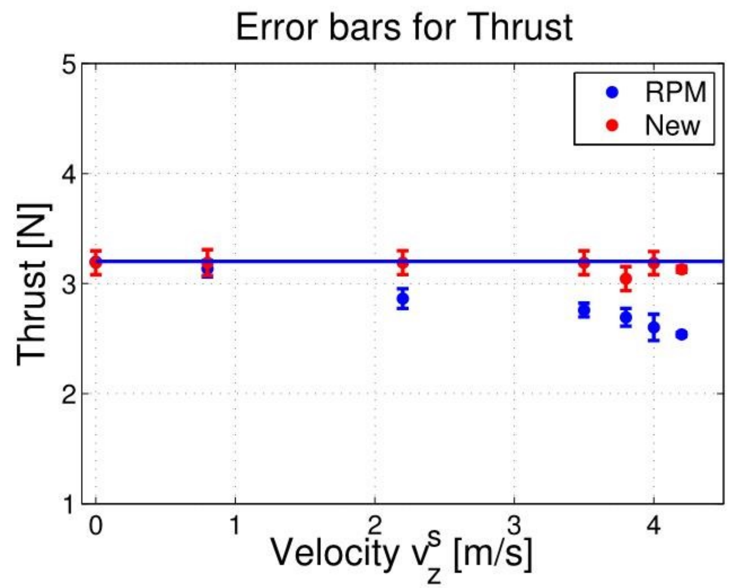
- Inner loop current control: tuned to be nearly unstable to generate fast rise time.
- Intermediate loop: regulates aerodynamic mechanical power
- The outer-level: standard PI regulator for thrust.

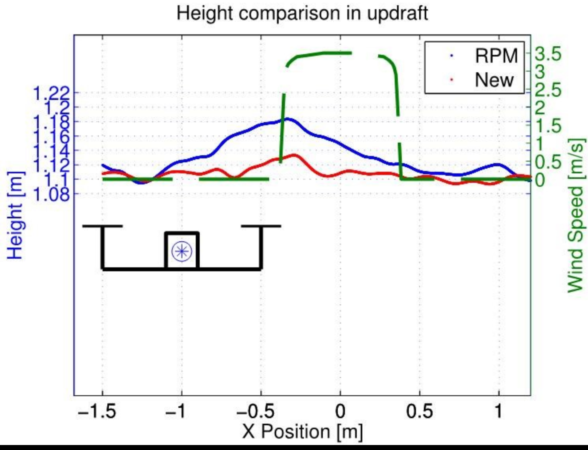
The resulting system has a rise time of 50ms (comparable to RPM controllers) and gust disturbance rejection of 90% (compared to RPM control).

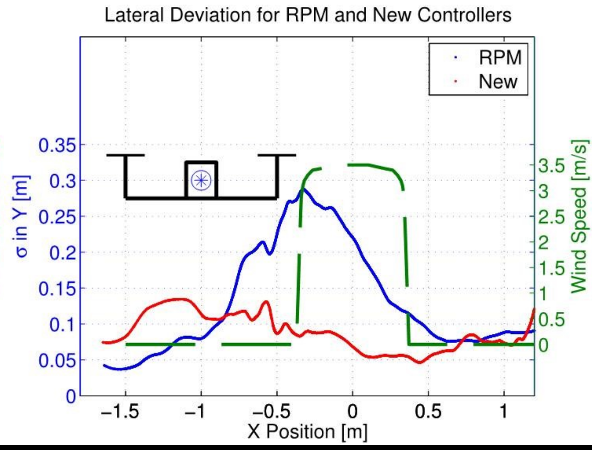
Thrust Control Experiments

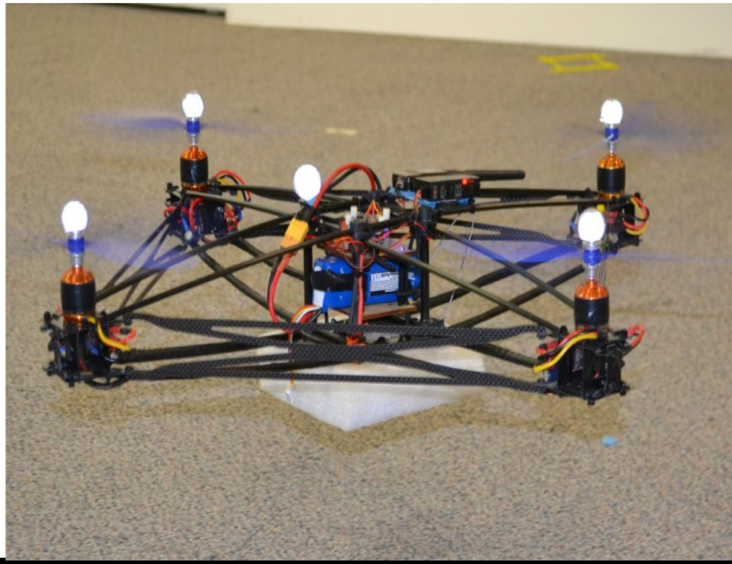


Using this approach – errors in thrust control for individual rotors can be improved by an order of magnitude: 20% error was reduced to <2% error.









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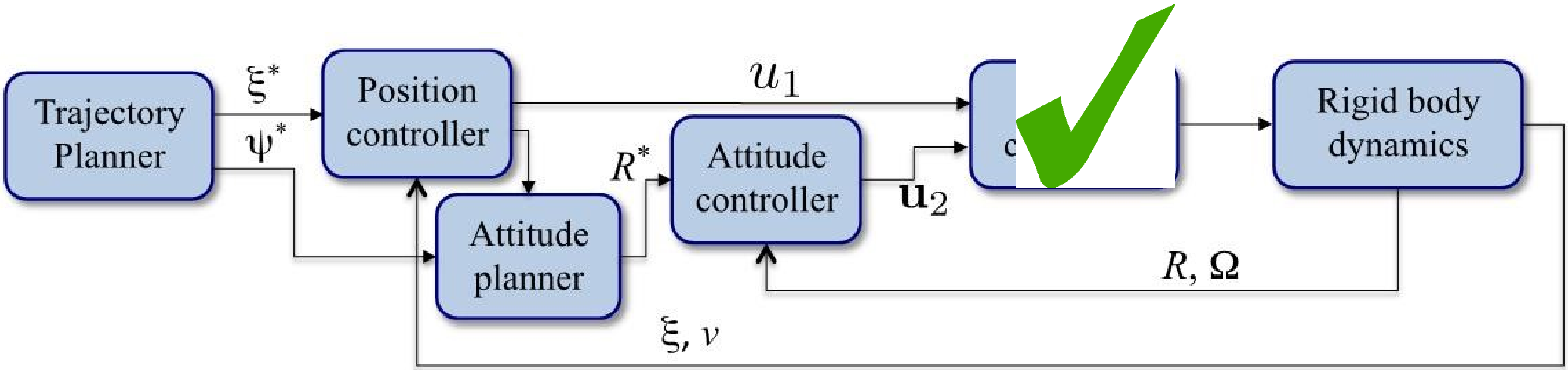


The key performance constraint for quadrotors is motor-rotor control response, including steady state error, disturbance rejection, and limited bandwidth.

Present systems typically have rise times around 50ms.

There isn't time in this talk to discuss this topic in detail today but this is an active research area.

Consider the goal of tracking a desired (feasible) trajectory $\boxed{(\xi^*(t), v^*(t), R^*(t), \Omega^*(t)).}$

Use full actuation of τ to track a set-point attitude trajectory $R^\star(t)$ along with its velocity $\Omega^\star(t)$ such that $\dot{R}^\star = R^\star \Omega^\star(t)_\times$.

Define a group error

$$\boxed{\tilde{R} = (R^\star)^\top R}$$

Canonical error for control

Driving $\tilde{R} \to I_3$ ensures that $R \to R^\star$.

The kinematics of the tracking error is given by

$$\begin{aligned}\dot{\tilde{R}} &= -\Omega^\star_\times \tilde{R} + \tilde{R}\Omega_\times = [\tilde{R}, \Omega^\star_\times] + \tilde{R}(\Omega_\times - \Omega^\star_\times) \\ &= [\tilde{R}, \Omega^\star_\times] + \tilde{R}\tilde{\Omega}_\times\end{aligned}$$

where $[A, B] = AB - BA$ is the matrix commutator and

$$\tilde{\Omega} := \Omega - \Omega^\star.$$

Assume $\Omega^\star$ and $\dot{\Omega}^\star$ are known, then define feedforward torque input

$$\boxed{\tau := I\dot{\Omega}^\star + \Omega^\star_\times I\Omega + \mathbf{u}_2}$$

Set $\tau := \tau^\star + \mathbf{u}_2$ then the error equations become

$$\dot{\tilde{R}} = [\tilde{R}, \Omega^\star_\times] + \tilde{R}\tilde{\Omega}_\times, \qquad I\dot{\tilde{\Omega}} = -\tilde{\Omega}_\times I\Omega + \mathbf{u}_2.$$

Choose proportional-derivative control action

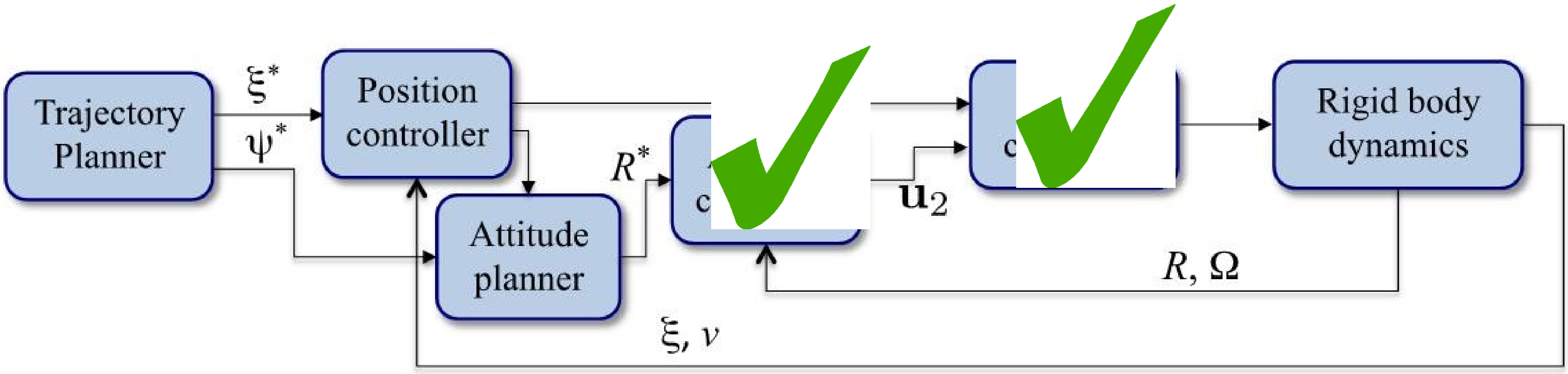
$$\mathbf{u}_2 = -\frac{k_P}{2}\operatorname{vrp}(\tilde{R} - \tilde{R}^\top) - k_D\tilde{\Omega}.$$

$$\boxed{\operatorname{vrp}(w_\times) = w}$$

Consider the Lyapunov function $\mathcal{L} := k_P \operatorname{tr}(\tilde{R}^\top \tilde{R}) + \frac{1}{2}\tilde{\Omega}^\top I \tilde{\Omega}$ and note that

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{L} := -k_D\|\tilde{\Omega}\|^2.$$

Applying Barbalats criteria one proves $\tilde{R} \to I_3$



The inner loop stability problem can be solved using estimates provided by the attitude filter discussed earlier in the talk.

This control and estimator run on the embedded avionics system at rates of 50-200Hz.

Trajectory stabilisation (position control)

Assume that there is a full feasible (**planned**) trajectory available

$$(\xi^p, \dot{\xi}^p, R^p, \Omega^p, \dot{\Omega}^p, u_1^p, u_2^p)$$

The linear trajectory goal $\xi^\star = \xi^p$ and $\dot{\xi}^\star = \dot{\xi}^p$ are used directly. However, actuation for the linear dynamics depends on the attitude

$$\dot{\xi} = v,$$

$$m\dot{v} = m\ddot{\xi} = mge_3 + u_1, \qquad \boxed{u_1 = TRe_3 \in \mathbb{R}^3}$$

Thus, assigning a control for ξ is defining the set point $R^\star$ on-line.

Choose control input

$$\boxed{u_1 = m\left(ge_3 + \ddot{\xi}^\star + k_d(\dot{\xi}^\star - \dot{\xi}) + k_p(\xi^\star - \xi)\right),}$$

such that

$$(\ddot{\xi}^\star - \ddot{\xi}) + k_d(\dot{\xi}^\star - \dot{\xi}) + k_p(\xi^\star - \xi) = 0$$

then $\xi(t) \to \xi^\star(t)$.

Note that specifying u_1 to define desired linear dynamics imposes a constraint on $R^\star$

$$\boxed{T^\star R^\star e_3 = m\left(g e_3 + \ddot{\xi}^\star + k_d(\dot{\xi}^\star - \dot{\xi}) + k_p(\xi^\star - \xi)\right),}$$

which is the online block for specifying the attitude tracking.

In order that the linear dynamics control is implemented, the attitude control must first regulate to the correct value. *time scale separation.*

In fact, since the attitude control is asymptotically stable, then regardless of loop gain — the linear error dynamics will eventually converge.

At least as long as

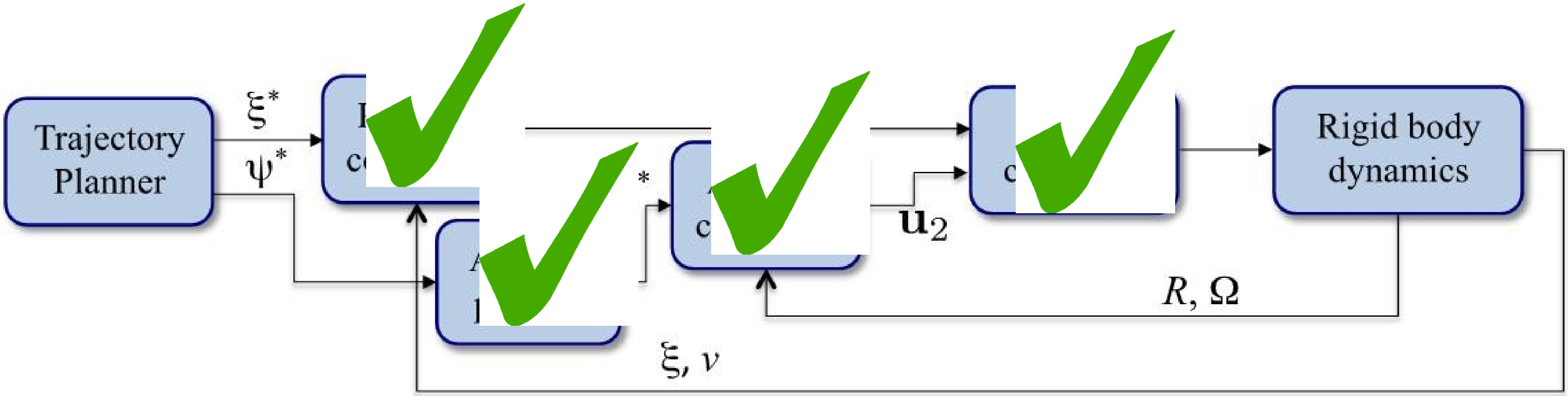
$$\dot{R}^\star \approx R^\star \Omega^\star$$

Needed for feedforward in attitude control

In practice, I use $R^\star$ defined by the control but use the planned trajectory for the higher order derivatives

$$\Omega^\star := \Omega^p, \quad \dot{\Omega}^\star := \dot{\Omega}^p$$

Control architecture



The trajectory planner should take into account the system limitations – in particular the rate limitations of the motor system.

Typically this is done by minimizing a suitable cost functional that penalizes the *snap* (or 4th derivative the trajectory).

Although quadrotors are underactuated they are differentially flat (or at worst almost flat) and this can be used to plan flat trajectories.

Let $\xi^p(t)$ denote a desired tractory. Then $v^p := \dot{\xi}^p$ is the desired velocity.

Moreover,

$$\boxed{\dot{v}^p = \frac{1}{m} T^p R^p e_3 + g e_3}$$

That is

$$\boxed{T^p := m \| \dot{v}^p - m g e_3 \|, \quad R^p e_3 := \frac{1}{T^p} m \dot{v}^p - m g e_3}$$

which fixes 2 degrees of freedom of the rotation R^p. The third degree of liberty is the yaw rotation and can be chosen as desired.

Finally,

$$\boxed{\Omega^p_\times := (R^p)^\top \dot{R}^p, \quad \tau^p := I \dot{\Omega}^p + \Omega^p \times I \Omega^p}$$

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High performance trajectories can be desined by minimizing a cost functional

$$(\xi^p(t), \psi^p(t)) = \arg \min_{\xi(t),\psi(t)} \int_0^T L(\xi, v, a, R, \Omega, \tau, \dot{\tau}) dt$$

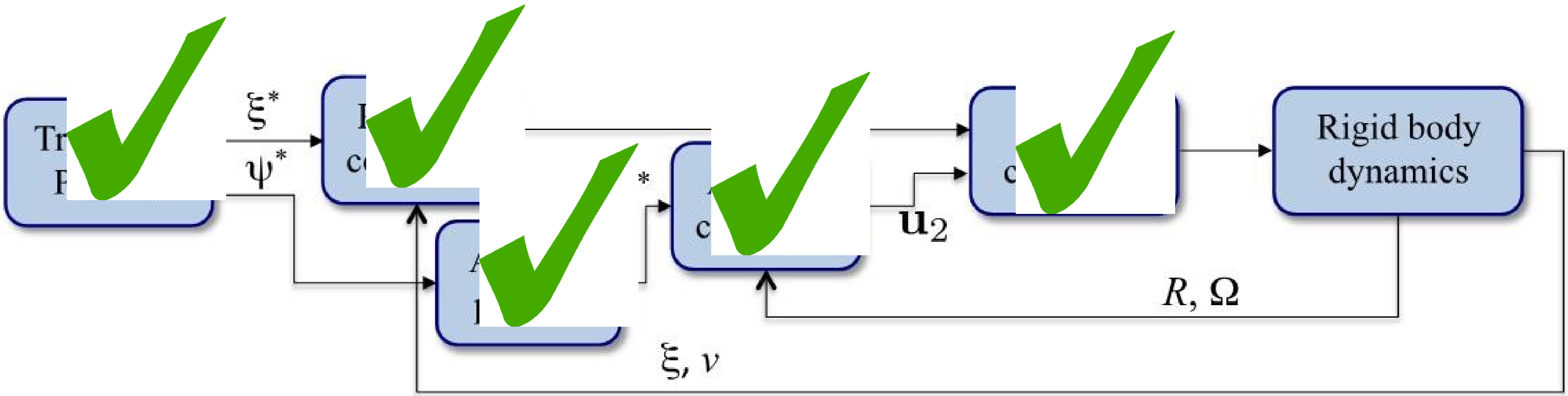
where ψ is the yaw parameter and the boundary conditions $\xi^p(0)$, $\xi^p(T)$, $\psi^p(0)$, $\psi^p(0)$ are given.

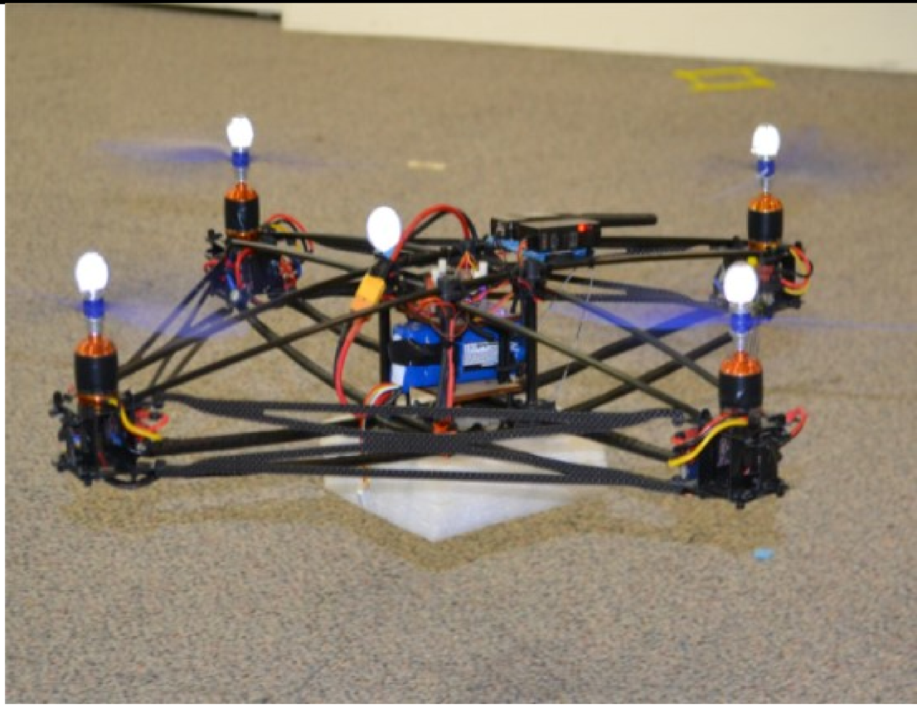
Minimum snap trajectories are obtained by minimizing a cost functional such as

$$\boxed{L(\xi, \dot{\xi}, \ddot{\xi}, \dddot{\xi}, \ddddot{\xi} \psi, \dot{\psi}, \ddot{\psi}) = (1-\alpha)(\ddddot{\xi})^4 + \alpha(\ddot{\psi})^2.}$$

for $\alpha > 0$. Minimum snap trajectories are important because they minimize the rate of actuation change required in the rotor/motor systems.

Control architecture



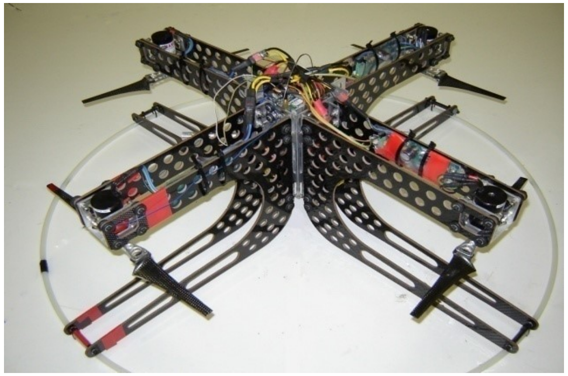


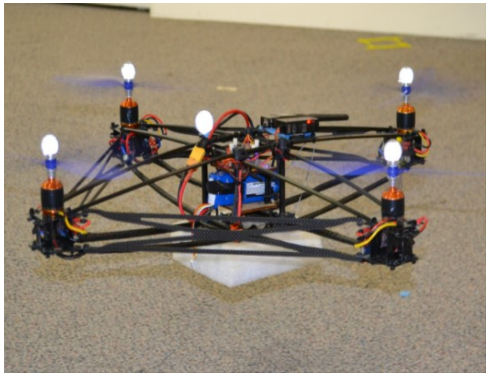
Fundamental questions in aerial robotic systems.

- Attitude estimation – IMU,
- Velocity estimation – GPS, IMU, VICON …
- Position estimation – Vision, VICON, GPS, …
- Motor control – embedded control.
- Attitude control – robustness and performance.
- Integrated path planning and position/velocity control – performance, replanning, feedback.

All of this has to implemented on embedded hardware weighing as little as possible.

Finally, you will want to do something with the quadrotor.





THANKS